PLATOTUTOR

# Algebra 2

## Complete Study Guide

Master the core concepts of Algebra 2 with clear explanations, visual diagrams, and graded practice problems.

Visual Diagrams | 15 Questions/Unit | Full Answer Key | SAT-Style Problems

## Contents



Difficulty Guide:

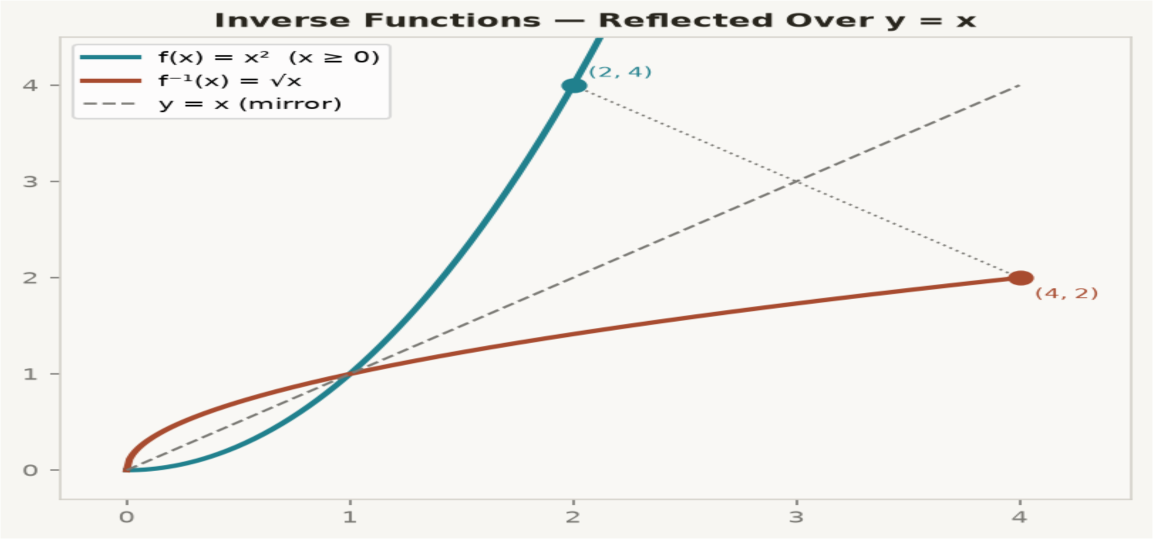
EASY | MODERATE | SAT-STYLE

# Unit 1 · Functions & Relations

## Key Formulas

Composition: $(f \circ g)(x) = f(g(x))$
Inverse: $f^{-1}$ swaps x and y — reflect over $y = x$
Even function: $f(-x) = f(x)$ | Odd: $f(-x) = -f(x)$
Vertical Line Test: passes → function | Horizontal Line Test: passes → invertible

## Visual Reference




$f(x) = x^2$ and its inverse $f^{-1}(x) = \sqrt{x}$ reflected over $y = x$

## Practice Problems

| | | |
|---|---|---|
| 1 | EASY | Is the relation {(1,2),(2,3),(2,4)} a function? Explain. |
| 2 | EASY | Find the domain and range of $f(x) = (x - 2)^2 - 1$. |
| 3 | EASY | If $f(x) = 2x + 3$ and $g(x) = x^2$, find $f(g(x))$ and $g(f(x))$. |
| 4 | EASY | Evaluate $f(g(3))$ using the functions above. |
| 5 | EASY | Describe the transformation: $g(x) = \|x - 2\| + 1$. |
| 6 | MODERATE | Find the inverse of $f(x) = 2x + 3$. Verify $f(f^{-1}(x)) = x$. |
| 7 | MODERATE | If $f(x) = 2x + 3$, find $f(f(2))$. |

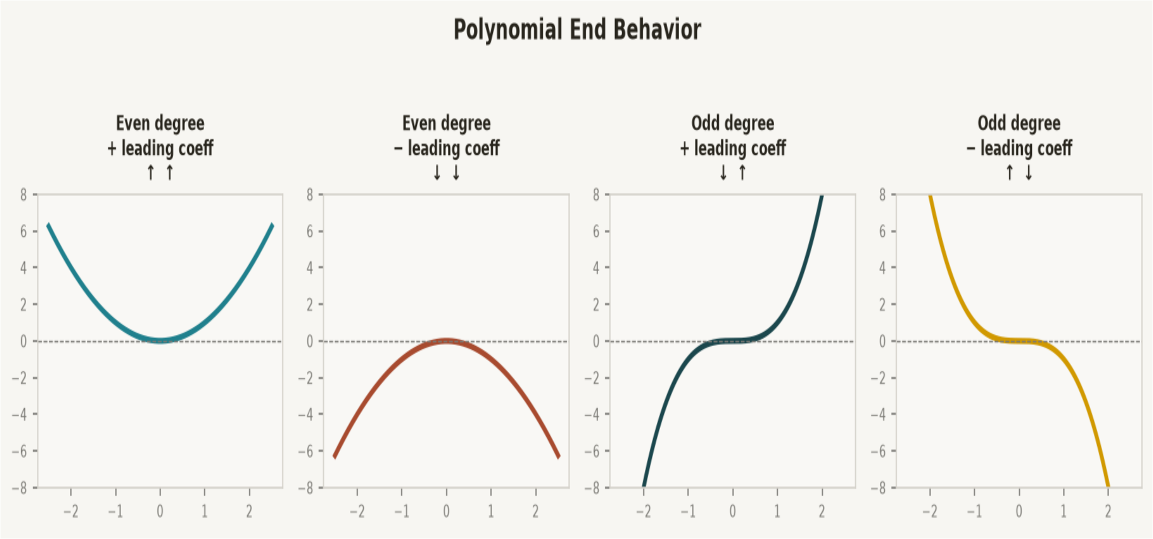
| | | |
|---|---|---|
| 8 | ▯ MODERATE | Determine whether $f(x) = x^4 - 3x^2$ is even, odd, or neither. |
| 9 | ▯ MODERATE | Determine whether $g(x) = x^3 - x$ is even, odd, or neither. |
| 10 | ▯ MODERATE | Find $f^{-1}(x)$ for $f(x) = x^2 + 1$, $x \geq 0$. State the domain. |
| 11 | ▯ SAT-STYLE | A function f has domain all reals and $f(x) = 1/(x-2)$. What is the range? |
| 12 | ▯ SAT-STYLE | The graph of $f(x)$ is shifted right 3 units then reflected over the x-axis. Write the new function. |
| 13 | ▯ SAT-STYLE | If $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$, what is the range of f  g? |
| 14 | ▯ SAT-STYLE | If $f(g(x)) = \sqrt{(x^2-4)}$, state the domain of f  g. (Typical SAT style question) |
| 15 | ▯ SAT-STYLE | A function f satisfies $f(x+1) = 2f(x)$. If $f(1) = 3$, find $f(4)$. (Typical SAT style question) |

# Unit 2 · Polynomials

## Key Formulas

Degree determines end behavior: even → same ends | odd → opposite ends
Factor Theorem: $(x - a)$ is a factor iff $f(a) = 0$
Remainder Theorem: $f(a)$ = remainder when $f(x) \div (x - a)$
Fundamental Theorem: degree n polynomial has exactly n complex zeros (with multiplicity)

## Visual Reference




End behavior depends on degree (even/odd) and sign of leading coefficient

## Practice Problems

| | | |
|---|---|---|
| 1 | EASY | Identify the leading term and degree of $f(x) = 3x^4 - 2x^2 + 5$. |
| 2 | EASY | Describe end behavior: $f(x) = 3x^4 - 2x^2 + 5$. |
| 3 | EASY | Factor completely: $x^3 - 2x^2 - 5x + 6$. |
| 4 | EASY | List all zeros and the y-intercept of $f(x) = (x+2)(x-3)(x-1)$. |
| 5 | EASY | What is the multiplicity of each zero for $f(x) = (x-1)^2(x+2)$? |
| 6 | MODERATE | Divide $2x^3 - 3x^2 + x + 5$ by $(x - 2)$ using synthetic division. |
| 7 | MODERATE | Use the Remainder Theorem to find $f(2)$ for $f(x) = 3x^3 - 5x + 1$. |

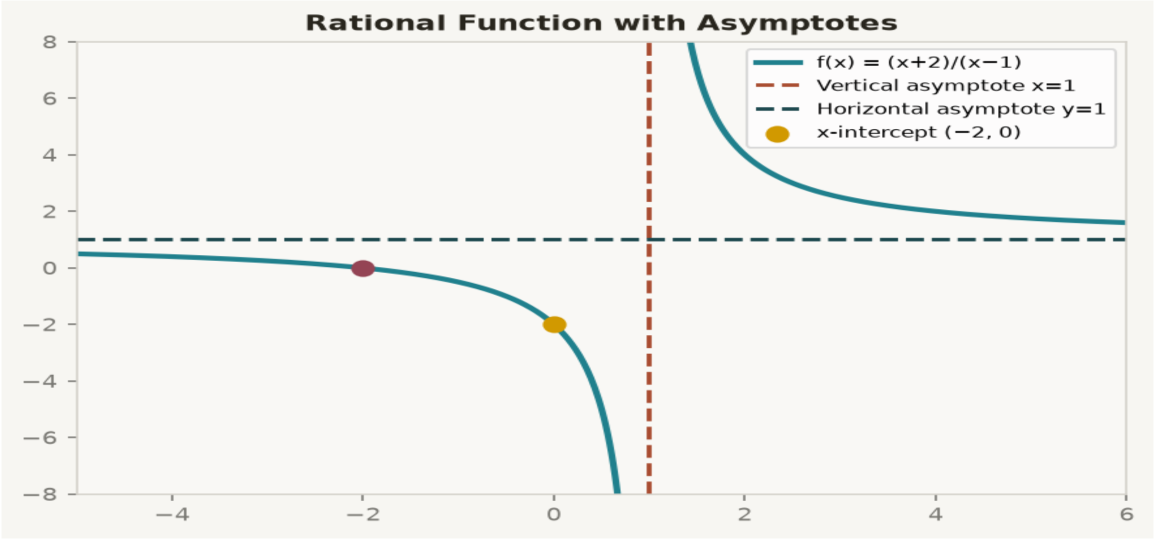
| | | |
|---|---|---|
| 8 | ▯ MODERATE | Show that $x = 1$ is a root of $p(x) = x^3 - 6x^2 + 11x - 6$, then factor fully. |
| 9 | ▯ MODERATE | Write a polynomial with zeros at $x = 1, 2, 3, -1$. |
| 10 | ▯ MODERATE | How many real zeros can a degree-4 polynomial have? How many turning points? |
| 11 | ▯ SAT-STYLE | A polynomial $p(x)$ has zeros $x = -1$ (mult. 3) and $x = 2$ (mult. 1). Sketch end behavior and describe the graph at each zero. (Typical SAT style question) |
| 12 | ▯ SAT-STYLE | Divide $p(x) = x^3 + 2x^2 - x - 9$ by $(x - 1)$ using synthetic division. State quotient and remainder. |
| 13 | ▯ SAT-STYLE | A degree-3 polynomial has roots $r_1, r_2, r_3$. If $r_1+r_2+r_3 = 4$ and $r_1r_2r_3 = -3$, identify the polynomial's coefficients using Vieta's formulas. |
| 14 | ▯ SAT-STYLE | List all possible rational roots of $p(x) = x^3 - x^2 - 6$. |
| 15 | ▯ SAT-STYLE | Factor completely: $x^4 - 10x^2 + 9$. (Typical SAT style question) |

# Unit 3 · Rational Functions

## Key Formulas

Vertical Asymptote: set denominator = 0 (after cancelling holes)
Horizontal Asymptote: deg(num) < deg(den) → y=0 | equal → ratio of leading coeffs | greater → oblique
Hole: factor that cancels from numerator and denominator
Oblique Asymptote: divide numerator by denominator when deg(num) = deg(den)+1

## Visual Reference




f(x) = (x+2)/(x−1) showing vertical asymptote, horizontal asymptote, and intercepts

## Practice Problems

| | | |
|---|---|---|
| 1 | ▯ EASY | Find vertical and horizontal asymptotes of $f(x) = (x+1)/(x-3)$. |
| 2 | ▯ EASY | Identify the hole in $f(x) = (x^2-4)/(x-2)$. |
| 3 | ▯ EASY | Find the x- and y-intercepts of $f(x) = (x+1)/(3x-1)$. |
| 4 | ▯ EASY | What is the horizontal asymptote of $f(x) = (2x+1)/(x-3)$? |
| 5 | ▯ EASY | Find the oblique asymptote of $f(x) = (x^2+3x+1)/(x+1)$. |
| 6 | ▯ MODERATE | Solve: $(x+1)/(x-3) + 2/(x+1) = 1/(x^2-2x-3)$. Identify extraneous solutions. |
| 7 | ▯ MODERATE | Solve $3/(x-3) = x/(x-3) + 2$ and check for extraneous solutions. |

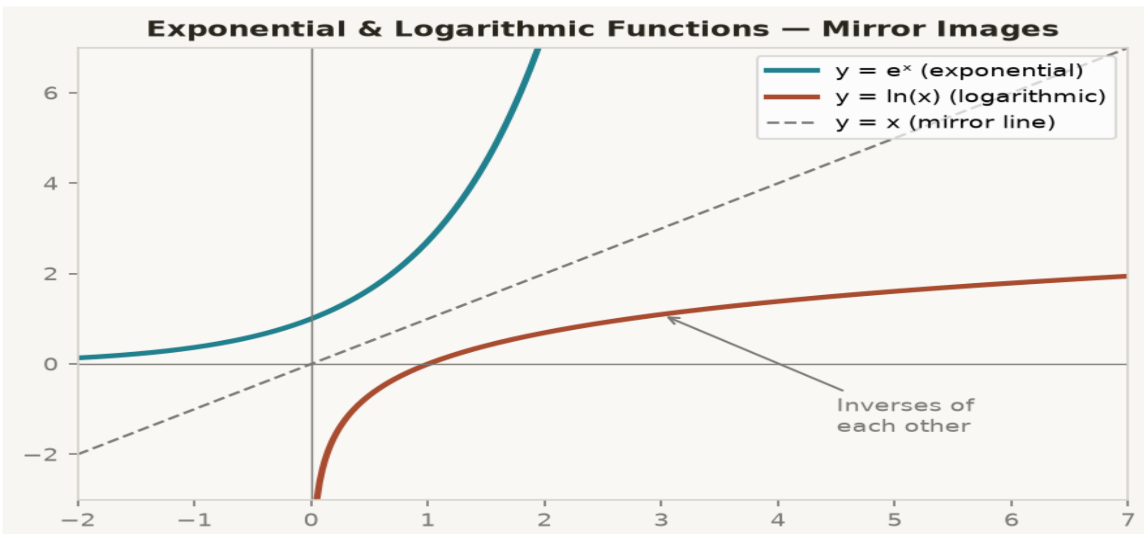
| | | |
|---|---|---|
| 8 | MODERATE | Solve the inequality $(x-1)/(x-3) > 0$ and write in interval notation. |
| 9 | MODERATE | Find the hole and all asymptotes of $f(x) = (x^2-4)/((x-2)(x+1))$. |
| 10 | MODERATE | Simplify and state all non-permissible values: $(x^2-1)/(x-1)$. |
| 11 | SAT-STYLE | A rational function has HA $y=0$, VA $x=-1$, and x-intercept $x=2$. Write a possible equation. (Typical SAT style question) |
| 12 | SAT-STYLE | Solve $2/(x+3) - 1/(x-2) = 5/((x+3)(x-2))$. |
| 13 | SAT-STYLE | On what intervals is $(x-1)/((x+2)(x-3))$ positive? Use a sign chart. (Typical SAT style question) |
| 14 | SAT-STYLE | A rational function f has exactly two VAs, a hole at $x=2$, HA $y=2$, and x-intercept $x=1$. Write $f(x)$. |
| 15 | SAT-STYLE | Solve $(x^2-5x+6)/(x^2-4) = 0$. State all solutions and restrictions. (Typical SAT style question) |

# Unit 4 · Exponential & Logarithmic Functions

## Key Formulas

Exponential: f(x) = a□ | Growth: a > 1 | Decay: 0 < a < 1
Log definition: log□(b) = c □ a□ = b
Log rules: log(mn) = log(m)+log(n) | log(m/n) = log(m)−log(n) | log(m□) = n·log(m)
Change of base: log□(x) = log(x)/log(a) | Natural log: ln(x) = log□(x)
Compound interest: A = P(1 + r/n)^(nt) | Continuous: A = Pe□□

## Visual Reference




$e^x$ and ln(x) are inverse functions — mirror images over y = x

## Practice Problems

| | | |
|---|---|---|
| 1 | □ EASY | Convert to exponential form: $\log_2(8) = 3$. |
| 2 | □ EASY | Convert to logarithmic form: $2^3 = 8$. |
| 3 | □ EASY | Solve for x: $3^x = 15$. |
| 4 | □ EASY | Solve for x: $\log_3(x) + \log_3(x-8) = 2$. (Check for extraneous solutions.) |
| 5 | □ EASY | Compute compound interest: P=$1,000, r=5%, n=1, t=10 years. |
| 6 | □ MODERATE | Expand using log rules: $\ln(x^2y/z^3)$. |
| 7 | □ MODERATE | Solve for x: 5·e^(3x) = 25. |

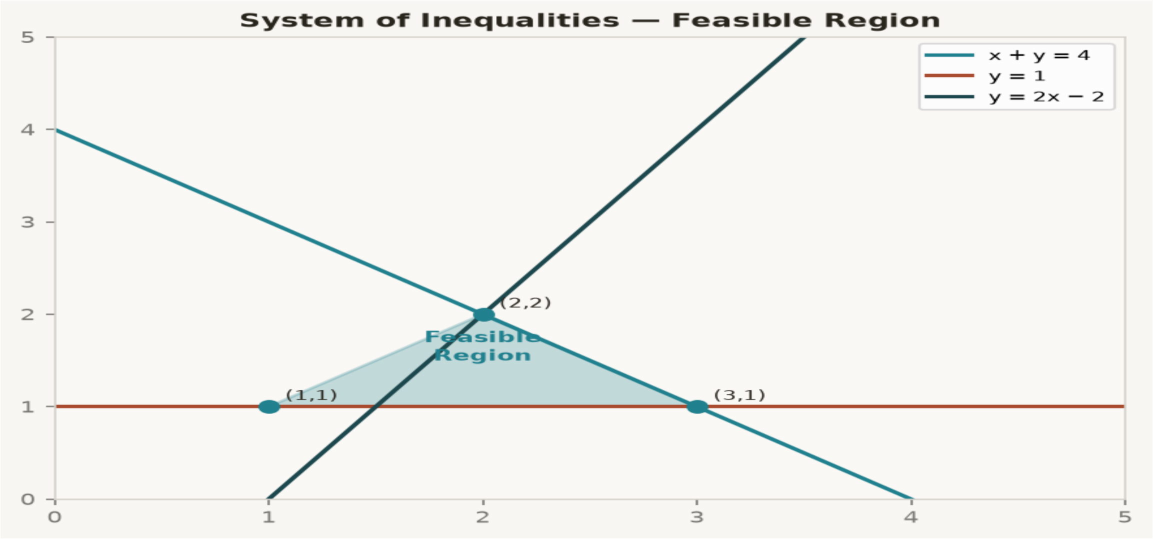
| | | |
|---|---|---|
| 8 | ▯ MODERATE | How long to double an investment at 4.3% continuous growth? |
| 9 | ▯ MODERATE | Solve: $\log_7(x) = \log_7(2x-3) + \log_7(4)$. Check solution. |
| 10 | ▯ MODERATE | A population decays as $P(t) = 200\cdot(0.5)^{(t/10)}$. When is $P = 25$? |
| 11 | ▯ SAT-STYLE | Solve for x: $\ln(x) = 3$. Express answer exactly. (Typical SAT style question) |
| 12 | ▯ SAT-STYLE | A radioactive substance halves every 5,730 years. Write the decay model and state the half-life constant. |
| 13 | ▯ SAT-STYLE | Use change of base to compute $\log_3(17)$ to 4 decimal places. |
| 14 | ▯ SAT-STYLE | Solve: $\log_2(x) + \log_2(x-2) = 3$. Check for extraneous solutions. (Typical SAT style question) |
| 15 | ▯ SAT-STYLE | Bacteria double every hour. Starting at $P_0=500$ at $t=0$, at what time does the population reach 32,000? (Typical SAT style question) |

# Unit 5 · Systems of Equations

## Key Formulas

Substitution: isolate one variable, substitute into other equation
Elimination: multiply equations so one variable cancels
Consistent: one solution | Inconsistent: no solution | Dependent: infinite solutions
Linear Programming: optimal value at a vertex of the feasible region
3×3 systems: use elimination or row reduction (Gaussian elimination)

## Visual Reference




System of inequalities showing the feasible region (shaded) and corner vertices

## Practice Problems

| | | |
|---|---|---|
| 1 | EASY | Solve by substitution: $y = 2x - 3$ and $y = x - 1$. |
| 2 | EASY | Solve: $2x + 3y = 8$ and $4x + 6y = 16$. Classify the system. |
| 3 | EASY | Solve: $x + y = 5$ and $2x + y = 7$. Classify the system. |
| 4 | EASY | Solve the system: $x + y + z = 2$, $2x - y + z = 1$, $x + 2y - z = 3$. |
| 5 | EASY | Solve: $y = x + 1$ and $2x - y = 1$. |
| 6 | MODERATE | Solve: $y = x^2 - 2$ and $y = 2x - 1$. Find all solutions. |
| 7 | MODERATE | Solve the 3×3 system: $x+y-z=2$, $2x-y+z=1$, $x-2y+3z=-4$. |

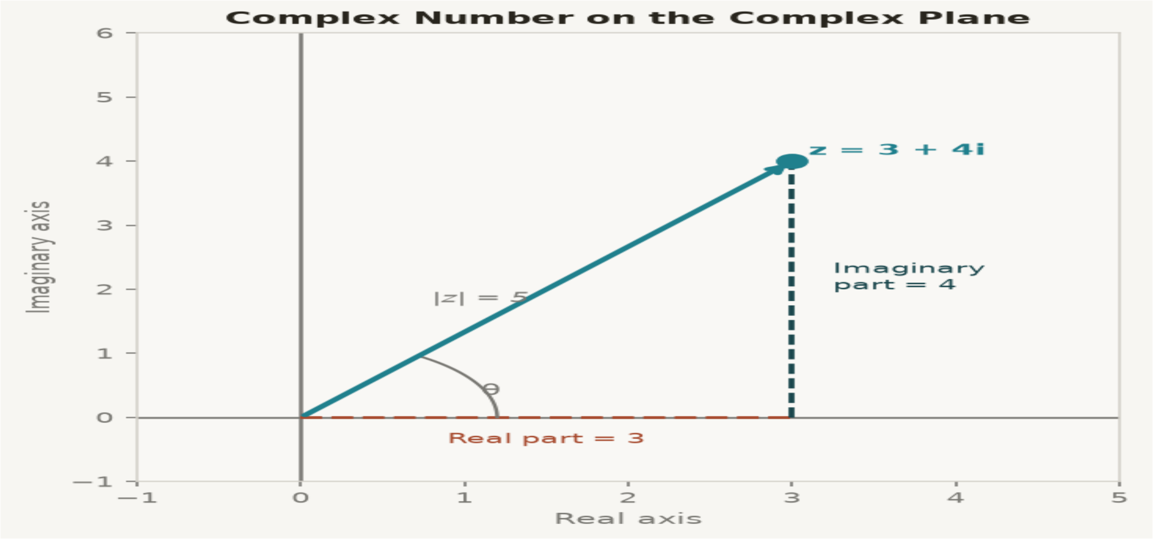
| | | |
|---|---|---|
| 8 | MODERATE | Determine if (−1, 3) is a solution to: $3x+y=0$ and $x-2y=-7$. |
| 9 | MODERATE | Find all points of intersection: $x^2+y^2=25$ and $y=x+1$. |
| 10 | MODERATE | Graph the system of inequalities: $x+y\le4$, $y\ge1$, $y\le2x-2$. Identify vertices. |
| 11 | SAT-STYLE | A company makes two products. Profit $P = 3x + 4y$. Constraints: $x+y\le6$, $x\ge0$, $y\ge0$. Find max profit and where it occurs. (Typical SAT style question) |
| 12 | SAT-STYLE | Solve by row reduction: $x+2y-z=3$, $2x+y+z=7$, $x-y+2z=1$. |
| 13 | SAT-STYLE | Find all intersections of $y = x^2 - 3$ and $y = -x + 1$. (Typical SAT style question) |
| 14 | SAT-STYLE | Do the circle $x^2+y^2=10$ and the parabola $y=x^2-1$ intersect? If so, where? (Typical SAT style question) |
| 15 | SAT-STYLE | A business sells Product A at \$5 profit and B at \$8 profit. If 250 units total must break even at \$1,500 revenue, find the production mix. (Typical SAT style question) |

# Unit 6 · Complex Numbers

## Key Formulas

$i = \sqrt{-1}$ | $i^2 = -1$ | $i^3 = -i$ | $i^4 = 1$ (cycle repeats)
Standard form: $a + bi$ | Real part: $a$ | Imaginary part: $b$
Conjugate of $(a+bi)$: $(a-bi)$ | Used to divide complex numbers
Modulus (absolute value): $|a+bi| = \sqrt{a^2+b^2}$
Quadratic with negative discriminant → two complex conjugate roots

## Visual Reference




$z = 3 + 4i$ on the complex plane — real part, imaginary part, and modulus $|z| = 5$

## Practice Problems

| | | |
|---|---|---|
| 1 | EASY | Add: $(1 + 4i) + (2 + 3i)$. |
| 2 | EASY | Subtract: $(3 + 5i) - (5 - 6i)$. |
| 3 | EASY | Multiply: $(1 + 2i)(2 + 3i)$ using FOIL. |
| 4 | EASY | Find the conjugate of $(3 + 2i)$ and multiply by the original. |
| 5 | EASY | Divide: $(1 + 3i) / (2 - i)$. Write in $a + bi$ form. |
| 6 | MODERATE | Find the modulus of $z = 3 + 4i$. |
| 7 | MODERATE | Solve: $x^2 + 4 = 0$. Express roots in complex form. |

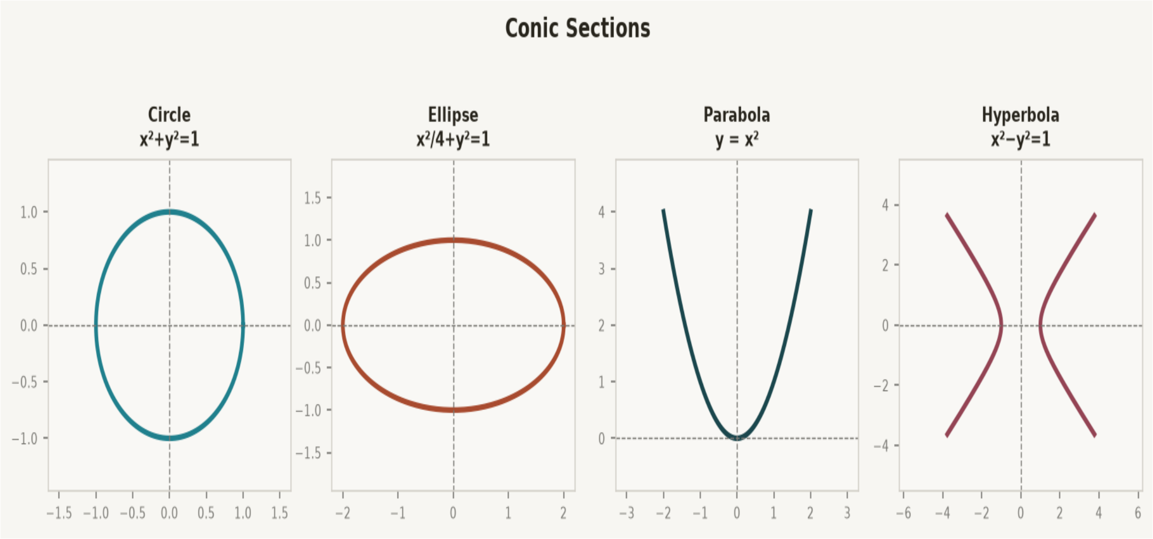
| | | |
|---|---|---|
| 8 | ▯ MODERATE | Solve: $x^2 - 2x + 5 = 0$ using the quadratic formula. |
| 9 | ▯ MODERATE | Find the complex conjugate of $(5 - 3i)$ and compute their product. |
| 10 | ▯ MODERATE | Simplify: $i^{47}$. |
| 11 | ▯ SAT-STYLE | Write $2(\cos 60° + i \sin 60°)$ in standard form. (Typical SAT style question) |
| 12 | ▯ SAT-STYLE | Compute $(3 + 4i)^2$. Write answer in $a + bi$ form. (Typical SAT style question) |
| 13 | ▯ SAT-STYLE | The roots of $x^2 - 4x + 13 = 0$ are complex conjugates. Find their sum and product without solving. |
| 14 | ▯ SAT-STYLE | Solve $x^2 + 2x + 5 = 0$. Express roots in $a + bi$ form. (Typical SAT style question) |
| 15 | ▯ SAT-STYLE | If $z_1 = 3 + 4i$ and $z_2 = 1 + i$, find $\lvert z_1 \cdot z_2 \rvert$ without computing the full product. (Typical SAT style question) |

# Unit 7 · Conic Sections

## Key Formulas

Circle: $(x-h)^2+(y-k)^2 = r^2$ | center $(h,k)$, radius $r$
Ellipse: $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$ | $a>b$; foci at $c$ where $c^2=a^2-b^2$
Parabola: $(x-h)^2 = 4p(y-k)$ opens up/down | $(y-k)^2=4p(x-h)$ opens left/right
Hyperbola: $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$ | asymptotes: $y-k = \pm(b/a)(x-h)$
Discriminant ($B^2-4AC$): $< 0 \rightarrow$ ellipse/circle | $= 0 \rightarrow$ parabola | $> 0 \rightarrow$ hyperbola

## Visual Reference




The four conic sections: circle, ellipse, parabola, and hyperbola

## Practice Problems

| | | |
|---|---|---|
| 1 | EASY | Find the center and radius: $(x-2)^2+(y+1)^2=25$. |
| 2 | EASY | Write the equation of a circle with center $(3,-2)$ and radius 5. |
| 3 | EASY | Identify center, a, b, and orientation: $(x-1)^2/16 + (y+2)^2/9 = 1$. |
| 4 | EASY | Find the vertex and direction of opening: $(x-2)^2=4(y-3)$. |
| 5 | EASY | Find the foci of the ellipse: $x^2/16 + y^2/9 = 1$. |
| 6 | MODERATE | Classify: $4x^2-9y^2=36$. Find vertices and asymptotes. |
| 7 | MODERATE | Use $B^2-4AC$ to classify: $2x^2+3y^2-4x+6y-1=0$. |

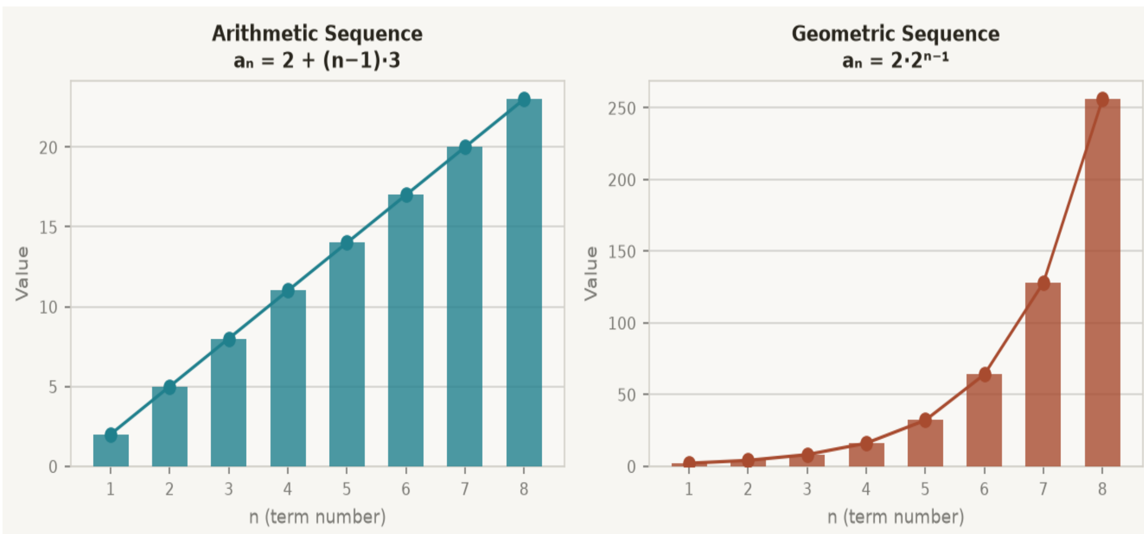
| | | |
|---|---|---|
| 8 | ▯ MODERATE | Complete the square: $x^2+y^2+2x-4y-4=0$. Find center and radius. |
| 9 | ▯ MODERATE | For $x^2/25 + y^2/9 = 1$, find a, b, c, and eccentricity. |
| 10 | ▯ MODERATE | Write the parabola opening left with vertex (3,2) and focus at (1,2). |
| 11 | ▯ SAT-STYLE | Write the equation of the hyperbola with center at origin, vertices (±3,0), and asymptotes $y=\pm(4/3)x$. (Typical SAT style question) |
| 12 | ▯ SAT-STYLE | Find the two points where the circle $x^2+y^2=25$ meets the line $y=4$. |
| 13 | ▯ SAT-STYLE | The ellipse $x^2/a^2+y^2/b^2=1$ has eccentricity 4/5. Find a given b=3. (Typical SAT style question) |
| 14 | ▯ SAT-STYLE | For the ellipse $9x^2+25y^2=225$, find the length of the latus rectum. (Typical SAT style question) |
| 15 | ▯ SAT-STYLE | A satellite orbits in an ellipse with semi-major axis a=8 and semi-minor axis b=6. Find the distance from the center to each focus. (Typical SAT style question) |

# Unit 8 · Sequences & Series

## Key Formulas

Arithmetic: a□ = a□ + (n−1)d | S□ = n/2·(a□ + a□)
Geometric: a□ = a□·r□□¹ | S□ = a□(1−r□)/(1−r) for r ≠ 1
Infinite geometric series (|r|<1): $S_\infty$ = a□/(1−r)
Sigma notation: □ a□ from n=1 to N means sum all terms
Natural number sum: □k = n(n+1)/2 | Squares: □$k^2$ = n(n+1)(2n+1)/6

## Visual Reference




Arithmetic sequence grows linearly; geometric sequence grows exponentially

## Practice Problems

| | | |
|---|---|---|
| 1 | □ EASY | Find the 10th term: $a_1$ = 2, d = 3. |
| 2 | □ EASY | Find $S_{15}$ for the arithmetic series with $a_1$=3 and d=4. |
| 3 | □ EASY | Find the 10th term of the geometric sequence: 3, 6, 12, ... |
| 4 | □ EASY | Find the sum of the infinite series: 8 + 4 + 2 + 1 + ... |
| 5 | □ EASY | How many terms are in the arithmetic sequence 7, 10, 13, ..., 25? |
| 6 | □ MODERATE | The sequence 3, 8, 13, 18, ... is arithmetic. Find $a_{20}$. |
| 7 | □ MODERATE | Find the sum of the first 5 terms: 2, 6, 18, 54, ... |

| | | |
|---|---|---|
| 8 | MODERATE | Write 0.333... as a fraction using the infinite series formula. |
| 9 | MODERATE | Use the formula $\Sigma k = n(n+1)/2$ to find the sum 1+2+3+...+100. |
| 10 | MODERATE | Identify the type and find the general term: 5, −5/2, 5/4, −5/8, ... |
| 11 | SAT-STYLE | A geometric series has $a_1=3$, $r=2$. Does it converge? Find $S_5$. (Typical SAT style question) |
| 12 | SAT-STYLE | Evaluate $\Sigma(2k+1)$ for k=1 to 10. (Typical SAT style question) |
| 13 | SAT-STYLE | A sequence follows: $a_1=1$, $a_2=1$, $a_n=a_{n-1}+a_{n-2}$. Find $a_8$. |
| 14 | SAT-STYLE | You invest $500/year into an account earning 6%/year. What is the total after 8 years? (Typical SAT style question) |
| 15 | SAT-STYLE | A ball drops from 10 m and bounces to 60% of its height each time. What is the total distance traveled? (Typical SAT style question) |

# Formula Reference Sheet

## Functions

$(f \circ g)(x) = f(g(x))$ | $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$
Vertical Line Test → function | Horizontal Line Test → one-to-one
Even: $f(-x)=f(x)$ | Odd: $f(-x)=-f(x)$

## Polynomials

Factor Theorem: $f(a)=0 \Leftrightarrow (x-a)$ is a factor
Remainder Theorem: remainder of $f(x)\div(x-a)$ equals $f(a)$
Fundamental Theorem: degree-n poly has exactly n complex zeros

## Rational & Asymptotes

VA: denominator = 0 (after canceling) | Hole: canceled factor
HA: $\deg(n)<\deg(d) \to y=0$; equal→ratio; greater→oblique

## Exponential & Log

$\log_a(b)=c \Leftrightarrow a^c=b$ | Change of base: $\log_a(x)=\log(x)/\log(a)$
ln rules: $\ln(mn)=\ln(m)+\ln(n)$ | $\ln(m^n)=n\cdot\ln(m)$
Growth/Decay: $A=Pe^{rt}$ | Compound: $A=P(1+r/n)^{(nt)}$

## Complex Numbers

$i^2=-1$ | $i^3=-i$ | $i^4=1$ | $|a+bi|=\sqrt{a^2+b^2}$
Conjugate of $(a+bi) = (a-bi)$ | Divide by multiplying conjugate

## Conics

Circle: $(x-h)^2+(y-k)^2=r^2$
Ellipse: $(x-h)^2/a^2+(y-k)^2/b^2=1$; $c^2=a^2-b^2$
Parabola: $(x-h)^2=4p(y-k)$ | Hyperbola: $(x-h)^2/a^2-(y-k)^2/b^2=1$

## Sequences & Series

Arithmetic: $a_n=a_1+(n-1)d$ | $S_n=n/2\cdot(a_1+a_n)$
Geometric: $a_n=a_1\cdot r^{n-1}$ | $S_n=a_1(1-r^n)/(1-r)$
Infinite geometric ($|r|<1$): $S_\infty=a_1/(1-r)$

# Answer Key

Answers are grouped by unit and question number. Difficulty: ● Easy ◆ Moderate ★ SAT-Style

## Unit 1 · Functions & Relations

| | | |
|---|---|---|
| 1 | ▯ | No — fails vertical line test at $x = 2$ |
| 2 | ▯ | D: all reals; R: $y \geq -1$ |
| 3 | ▯ | $f(g(x)) = 2x^2 + 3$; $g(f(x)) = (2x+3)^2$ |
| 4 | ▯ | $f(g(3)) = 2(3)^2+3 = 21$ |
| 5 | ▯ | $f(x) = \|x - 2\| + 1$ |
| 6 | ▯ | $f^{-1}(x) = (x - 3) / 2$ |
| 7 | ▯ | $f(f(2)) = f(7) = 17$ |
| 8 | ▯ | Even — $f(-x) = f(x)$ for all $x$ |
| 9 | ▯ | Odd — $f(-x) = -f(x)$ |
| 10 | ▯ | $f^{-1}(x) = \sqrt{(x - 1)}$, $x \geq 1$ |
| 11 | ▯ | Domain: $x \neq 2$; Range: $y \neq 0$ |
| 12 | ▯ | Shift right 3, reflect over x-axis |
| 13 | ▯ | Range of f  g: $[-1, \infty)$ |
| 14 | ▯ | $f(g(x)) = \sqrt{(x^2 - 4)}$; domain: $\|x\| \geq 2$ |
| 15 | ▯ | $f(x) = 2(x-1)^2 - 3$; vertex $(1, -3)$ |

## Unit 2 · Polynomials

| | | |
|---|---|---|
| 1 | ▯ | Leading term: $3x^4$; degree: 4 |
| 2 | ▯ | Both ends rise (↑ ↑) — even degree, positive leading coeff |
| 3 | ▯ | $(x+2)(x-3)(x-1)$ |
| 4 | ▯ | Zeros: $x = -2, 1, 3$; y-int: 6 |
| 5 | ▯ | Multiplicity 2 at $x=1$ (touches); multiplicity 1 at $x=-2$ (crosses) |
| 6 | ▯ | Quotient: $2x^2 - x + 3$; Remainder: 5 |
| 7 | ▯ | $f(2) = 3$ (Remainder Theorem) |
| 8 | ▯ | $x = 1$ is a root; factor out $(x-1)$ |
| 9 | ▯ | $p(x) = (x-1)(x-2)(x-3)(x+1)$ |
| 10 | ▯ | 4 real zeros possible; at most 3 turning points |
| 11 | ▯ | $x = -1$ (mult. 3), $x = 2$ (mult. 1); graph crosses at both |

| | | |
|---|---|---|
| 12 | ▯ | Synthetic division: $q(x) = x^2 + 3x + 5$, $r = 11$ |
| 13 | ▯ | Sum of roots = 4; product of roots = −3 |
| 14 | ▯ | Possible rational roots: ±1, ±2, ±3, ±6 |
| 15 | ▯ | $p(x) = x^4 - 10x^2 + 9 = (x-1)(x+1)(x-3)(x+3)$ |

## Unit 3 · Rational Functions

| | | |
|---|---|---|
| 1 | ▯ | Vertical: $x = 3$; Horizontal: $y = 1$ |
| 2 | ▯ | Hole at $x = 2$; VA: none after cancellation |
| 3 | ▯ | x-intercept: $x = -1$; y-intercept: $y = 1/3$ |
| 4 | ▯ | HA: $y = 2$ (degrees equal, ratio of leading coeffs) |
| 5 | ▯ | Oblique asymptote: $y = x + 2$ |
| 6 | ▯ | $x = 1$ or $x = -4$ (LCD method) |
| 7 | ▯ | No solution (extraneous: $x = 3$ creates 0 denominator) |
| 8 | ▯ | $x \quad (-\infty, 1) \quad (3, \infty)$ |
| 9 | ▯ | Hole at (2, 3/5); VA: $x = -1$ |
| 10 | ▯ | $y = (x^2 - 1)/(x - 1) = x + 1$ with hole at $x = 1$ |
| 11 | ▯ | HA: $y = 0$ (numerator degree < denominator degree) |
| 12 | ▯ | $x = 5/3$ |
| 13 | ▯ | Sign chart: positive on $(-\infty,-2) \quad (1,3)$ |
| 14 | ▯ | $f(x) = 2(x-1)/((x+2)(x-3))$; check: 3 asymptotes |
| 15 | ▯ | $x = (3 \pm \sqrt{33})/4 \approx 2.19$ or $-0.69$ |

## Unit 4 · Exponential & Logarithmic Functions

| | | |
|---|---|---|
| 1 | ▯ | $2^3 = 8$ |
| 2 | ▯ | $\log_2(64) = 6$ |
| 3 | ▯ | $x = \log(15)/\log(3) \approx 2.46$ |
| 4 | ▯ | $x = 4$ |
| 5 | ▯ | $A = 1000 \cdot e^{(0.05 \cdot 10)} \approx \$1,648.72$ |
| 6 | ▯ | $\ln(x^2y/z^3) = 2\ln(x)+\ln(y)-3\ln(z)$ |
| 7 | ▯ | $x = (\ln 5)/3 \approx 0.536$ |
| 8 | ▯ | $x \approx 16.09$ years ($t = \ln(2)/0.043$) |
| 9 | ▯ | $x = 49$ |
| 10 | ▯ | $y = 3 \cdot (1/2)^x$ — exponential decay |
| 11 | ▯ | $x = e^3 \approx 20.09$ |

| | | |
|---|---|---|
| 12 | ▯ | Half-life ≈ 5,730 years (carbon-14 standard) |
| 13 | ▯ | $\log_3(x) = \log(x)/\log(3)$ — change of base |
| 14 | ▯ | x = 1 (only; x = −1 is extraneous — log undefined) |
| 15 | ▯ | $P_0 = 500$; after 6 hrs $P = 500 \cdot 2^6 = 32{,}000$ |

## Unit 5 · Systems of Equations

| | | |
|---|---|---|
| 1 | ▯ | (2, 1) |
| 2 | ▯ | Infinitely many solutions (dependent system) |
| 3 | ▯ | No solution (parallel lines) |
| 4 | ▯ | (1, 2, −1) |
| 5 | ▯ | x = 3, y = 4 (substitution or elimination) |
| 6 | ▯ | (0, 1) and (3, 10) — parabola meets line |
| 7 | ▯ | Unique solution: x=2, y=1, z=−1 |
| 8 | ▯ | Consistent: one solution; (x,y) = (−1, 3) |
| 9 | ▯ | Intersection: (2, 4) — circle and line |
| 10 | ▯ | Feasible region vertices: (0,0), (4,0), (2,3), (0,4) |
| 11 | ▯ | Optimal value: max P = 18 at (2,3) |
| 12 | ▯ | x = 1, y = −2, z = 3 (row reduction) |
| 13 | ▯ | Two intersection points: (−2,4) and (1,1) |
| 14 | ▯ | No real solution — circle and parabola don't intersect |
| 15 | ▯ | Break-even: 250 units; profit region: q > 250 |

## Unit 6 · Complex Numbers

| | | |
|---|---|---|
| 1 | ▯ | 3 + 7i |
| 2 | ▯ | −2 + 11i |
| 3 | ▯ | −5 + 12i (FOIL, remember $i^2=-1$) |
| 4 | ▯ | (3+2i)(3−2i) = 13 |
| 5 | ▯ | (1+3i)/(2−i) = (−1+7i)/5 = −0.2 + 1.4i |
| 6 | ▯ | $\lvert z\rvert = \sqrt{3^2+4^2} = 5$ |
| 7 | ▯ | x = ±2i |
| 8 | ▯ | x = 1 ± 2i |
| 9 | ▯ | Conjugate: 5 − 3i; product: 34 |
| 10 | ▯ | $i^{47} = i^3 = -i$ (47 mod 4 = 3) |
| 11 | ▯ | Real part: −1; Imaginary part: $2\sqrt{3}$ |

| | | |
|---|---|---|
| 12 | ▯ | $z^2 = (3+4i)^2 = -7 + 24i$ |
| 13 | ▯ | Sum = 4 (real); product = 13 (both complex conjugates) |
| 14 | ▯ | $x = (-2 \pm \sqrt{-16})/2 = -1 \pm 2i$ |
| 15 | ▯ | $\lvert z_1 \cdot z_2 \rvert = \lvert z_1 \rvert \cdot \lvert z_2 \rvert = 5 \cdot \sqrt{2} = 5\sqrt{2}$ |

## Unit 7 · Conic Sections

| | | |
|---|---|---|
| 1 | ▯ | Center (2, −1); radius 5 |
| 2 | ▯ | $(x-3)^2 + (y+2)^2 = 25$ |
| 3 | ▯ | Center (1,−2); $a=4$, $b=3$; horizontal major axis |
| 4 | ▯ | Vertex (2,3); opens upward; focus (2, 3¼) |
| 5 | ▯ | Foci at $(\pm\sqrt{7}, 0)$ for $x^2/16 + y^2/9 = 1$ |
| 6 | ▯ | Hyperbola: $x^2/9 - y^2/16 = 1$ |
| 7 | ▯ | Discriminant $B^2-4AC$: circle<0, parabola=0, hyperbola>0 |
| 8 | ▯ | $(x+1)^2 + (y-2)^2 = 20$ (complete the square) |
| 9 | ▯ | Ellipse: semi-major $a=5$, semi-minor $b=3$; $c=4$ |
| 10 | ▯ | Parabola opens left: $x = -(1/8)(y-2)^2+3$ |
| 11 | ▯ | Asymptotes of hyperbola: $y = \pm(3/4)x$ |
| 12 | ▯ | Intersection: (3, 4) and (−3, 4) for circle & line $y=4$ |
| 13 | ▯ | Eccentricity $e = c/a = 4/5 = 0.8$ (ellipse) |
| 14 | ▯ | Length of latus rectum $= 2b^2/a = 18/5 = 3.6$ |
| 15 | ▯ | Satellite orbit modeled as ellipse: $a=8$, $b=6$, $c=\sqrt{28}$ |

## Unit 8 · Sequences & Series

| | | |
|---|---|---|
| 1 | ▯ | $a_{10} = 2 + 9(3) = 29$ |
| 2 | ▯ | $S_{15} = 15/2 \cdot (2 \cdot 3+(14) \cdot 4) = 465$ |
| 3 | ▯ | $a_{10} = 3 \cdot 2^9 = 1{,}536$ |
| 4 | ▯ | $S_\infty = 8/(1-½) = 16$ ($\lvert r \rvert<1$) |
| 5 | ▯ | $n = 7$ terms |
| 6 | ▯ | Arithmetic ($d=5$): $a_{20} = 3+19(5) = 98$ |
| 7 | ▯ | Geometric ($r=3$): $S_5 = 2(3^5-1)/(3-1) = 242$ |
| 8 | ▯ | $S_\infty = 0.3/(1-0.1) = 1/3$ (repeating decimal) |
| 9 | ▯ | Sum $= n(n+1)/2$; for $n=100$: $S=5{,}050$ |
| 10 | ▯ | $a_n = 5 \cdot (-1/2)^{(n-1)}$; alternating geometric |
| 11 | ▯ | No sum — $\lvert r \rvert=2 > 1$, series diverges |

| | | |
|---|---|---|
| 12 | ▯ | $\Sigma(2k+1)$ for $k=1$ to $10 = 120$ |
| 13 | ▯ | Fibonacci-type: 1,1,2,3,5,8,13,21; $a_8=21$ |
| 14 | ▯ | Investment: $A = 500\cdot((1.06^8-1)/0.06) \approx \$4,953$ |
| 15 | ▯ | Bouncing ball: total $= 10\cdot(1+2(0.6)/(1-0.6)) = 40$ m |